


Lesson 27: Nature of Solutions of a System of Linear Equations

2. $\left\{\begin{array}{l}7 x+2 y=-4 \\ x-y=5\end{array}\right.$
3. $\left\{\begin{array}{l}9 x+6 y=3 \\ 3 x+2 y=1\end{array}\right.$

To Use the substitution method:
Look for $x=$ $\square$

$$
y=\square
$$

$\rightarrow$ If you don't have $x=\square$ or $y=\square$, move the other terms away to isolate a letter

$$
\begin{aligned}
2 x+2 y & =4 \\
-2 x & -2 x \\
\frac{2 y}{2} y & =\frac{-2 x}{2}+\frac{4}{2} \\
y & =-x+2
\end{aligned}
$$

Determine the nature of the solution to each system of linear equations.
algebraically, and then verify that your solution is correct by graphing.
4. $\left\{\begin{array}{l}3 x+3 y=-21\end{array}\right.$


$$
\text { 3.) }\left\{\begin{array}{ll}
(y)=2 x+3 & \left(\begin{array}{l}
x, y \\
y=3 x-5
\end{array}\right. \\
y=19)
\end{array}\right] \begin{array}{rl}
3 x-5=2 x+3 & y=2 x+3 \\
-2 x & y=-2 x \\
x-\$=+5 & y=2(8)+3 \\
\hline x=8 & y=16+3 \\
\hline 5=19
\end{array}
$$

$$
\text { 4.) }\left\{\begin{array}{ll}
x+2(y)=5 \\
y=2 x-10
\end{array} \quad\left(\begin{array}{ll}
x & y \\
5,0
\end{array}\right), ~ y=2(5)-10\right\}
$$

6. $\left\{\begin{array}{l}x=12 y-4 \\ x=9 y+7\end{array}\right.$

$$
\begin{array}{cc}
\left\{\begin{array}{cc}
-2 x+(y)=-3 x \\
3 x+(y)=5
\end{array}\right. & y=-2 x+3 \\
\binom{x, y}{2,-1} \\
3 x+(-2 x+3)=5 & y=-2 x+3 \\
x+3=5 & y=-2(2)+3 \\
-\beta=-3 & y=-4+3 \\
x=2 & y=-1
\end{array}
$$

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Lesson Summary
A system of linear equations can have a unique solution, no solutions, or infinitely many solutions
Systems with a unique solution will be comprised of linear equations that have different slopes that graph as
distinct lines, intersecting at only one point
Systems with no solution will be comprised of linear equations that have the same slope that graph as parallel lines
(no intersection).
Systems with infinitely many solutions will be comprised of linear equations that have the same slope and y
Intercept that graph as the same line. When equations graph as the same line, every solution to one equation will
also be a solution to the other equation.
A system of linear equations can be solved using a substitution method. That is, if two expressions are equal to the
Asme value, then they can be written equal to one another.
Example
                    l}\begin{array}{l}{y=5x-8}\\{y=6x+3}
Since both equations in the system are equal to \(y\), we can write the equation, \(5 x-8=6 x+3\), and use it to solve for \(x\) and then the system.
Example:
\[
\left\{\begin{array}{l}
3 x=4 y+2 \\
x=y+5
\end{array}\right.
\]
Multiply each term of the equation \(x=y+5\) by 3 to produce the equivalent equation \(3 x=3 y+15\). As in the previous example, since both equations equal \(3 x\), we can write \(4 y+2=3 y+15\). This equation can be used to solve for \(y\) and then the system.
Problem Set
Determine the nature of the solution to each system of linear equations. If the system has a solution, find algebraically, and then verify that your solution is correct by graphing.
1. \(\left\{\begin{array}{l}y=\frac{3}{7} x-8 \\ 3 x-7 y=1\end{array}\right.\)
\(3 x-7 y=1\)
2. \(\left\{\begin{array}{l}2 x-5=y \\ -3 x-1=2 y\end{array}\right.\)
3. \(\left\{\begin{array}{l}x=6 y+7 \\ x=10 y+2\end{array}\right.\)
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4. $\left\{\begin{array}{l}5 y=\frac{15}{4} x+25 \\ y=\frac{3}{4} x+5\end{array}\right.$
5. $\left\{\begin{array}{l}x+9=y \\ x=4 y-6\end{array}\right.$
6. $\left\{\begin{array}{l}3 y=5 x-15 \\ 3 y=13 x-2\end{array}\right.$
7. $\left\{\begin{array}{c}6 x-7 y=\frac{1}{2}\end{array}\right.$
8. $\{5 x-2 y=6$
9. $\left\{\begin{array}{l}y=\frac{3}{2} x-6 \\ 2 y=7-4 x\end{array}\right.$
10. $\{7 x-10=y$
11. Write a system of linear equations with $(-3,9)$ as its solution.
