$y = \text{rate}\,x + \text{Extra}$

Lesson 9: Determining the Equation of a Line Fit to Data

$\frac{\Delta y}{\Delta x}$

Initial value

Classwork

Example 1: Crocodiles and Alligators

Scientists are interested in finding out how different species adapt to finding food sources. One group studied crocodilians to find out how their bite force was related to body mass and diet. The table below displays the information they collected on body mass (in pounds) and bite force (in pounds).

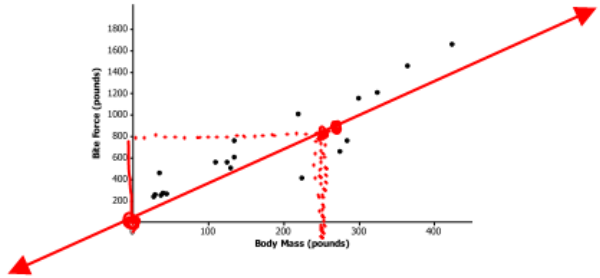
Crocodilian Biting

Species	Body Mass (pounds)	Bite Force (pounds)
Dwarf crocodile	35	450
Crocodile F	40	260
Alligator A	30	250
Caiman A	28	230
Caiman B	37	240
Caiman C	45	255
Crocodile A	110	550
Nile crocodile	275	650
Crocodile B	130	500
Crocodile C	135	600
Crocodile D	135	750
Caiman D	125	550
Indian gharial crocodile	225	400
Crocodile G	220	1,000
American crocodile	270	900
Crocodile E	285	750
Crocodile F	425	1,650
American alligator	300	1,150
Alligator B	325	1,200
Alligator C	365	1,450

Data Source: http://journals.plos.org/plosone/article?id=10.1371/journal.pone.0031781#pone-0031781-t001

(Note: Body mass and bite force have been converted to pounds from kilograms and newtons, respectively.)

As you learned in the previous lesson, it is a good idea to begin by looking at what a scatter plot tells you about the data. The scatter plot below displays the data on body mass and bite force for the crocodilians in the study.



Exercises 1–6

1. Describe the relationship between body mass and bite force for the crocodilians shown in the scatter plot.

positive

2. Draw a line to represent the trend in the data. Comment on what you considered in drawing your line.

3. Based on your line, predict the bite force for a crocodilian that weighs 220 pounds. How does this prediction compare to the actual bite force of the 220-pound crocodilian in the data set?

plug-in for x

$y = \text{rate} \cdot x + \text{Extra}$

$y = \frac{800}{250}x + 0$

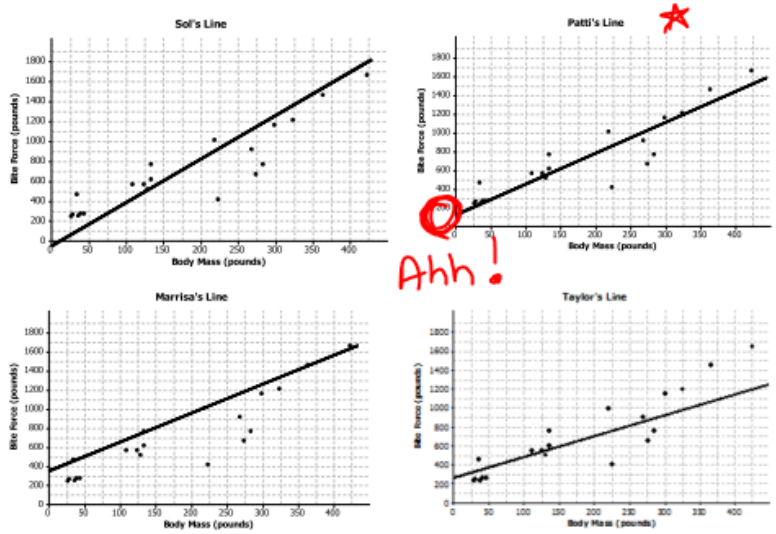
$y = 3.2x$

$y = 3.2(220)$

$= 704$ lbs of bite force

3.2 pounds of bite force per pound of body mass.

4. Several students decided to draw lines to represent the trend in the data. Consider the lines drawn by Sol, Patti, Marrisa, and Taylor, which are shown below.



For each student, indicate whether or not you think the line would be a good line to use to make predictions. Explain your thinking.

a. Sol's line

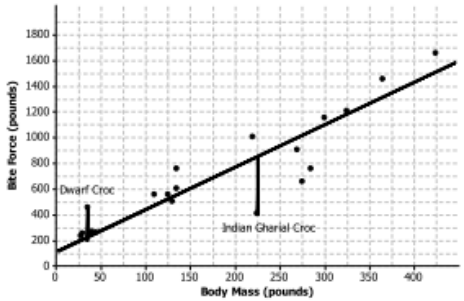
b. Patti's line

c. Marrisa's line

d. Taylor's line

5. What is the equation of your line? Show the steps you used to determine your line. Based on your equation, what is your prediction for the bite force of a crocodilian weighing 200 pounds?

6. Patti drew vertical line segments from two points to the line in her scatter plot. The first point she selected was for a dwarf crocodile. The second point she selected was for an Indian gharial crocodile.

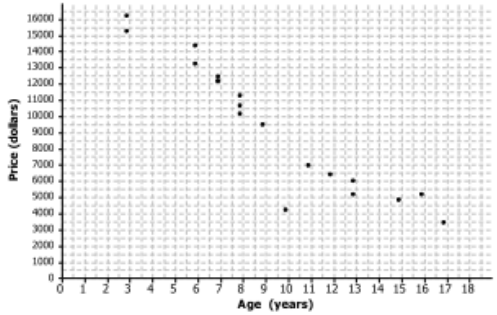


a. Would Patti's line have resulted in a predicted bite force that was closer to the actual bite force for the dwarf crocodile or for the Indian gharial crocodile? What aspect of the scatter plot supports your answer?

b. Would it be preferable to describe the trend in a scatter plot using a line that makes the differences in the actual and predicted values large or small? Explain your answer.

Exercise 7: Used Cars

7. Suppose the plot below shows the age (in years) and price (in dollars) of used compact cars that were advertised in a local newspaper.

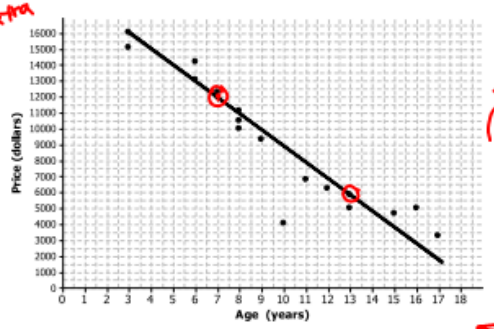


a. Based on the scatter plot above, describe the relationship between the age and price of the used cars.

As the age of a car increases, the price decreases.

b. Nora drew a line she thought was close to many of the points and found the equation of the line. She used the points $(13, 6000)$ and $(7, 12000)$ on her line to find the equation. Explain why those points made finding the equation easy.

$y = \frac{\Delta y}{\Delta x}x + \text{Extra}$



x	y
7	12,000
13	6,000

$-6,000$

$y = \frac{-6,000}{6}x +$

$y = -1,000x + 19,000$

The price goes down $1,000 per year. The $19,000 tells us the price of a brand new car.

c. Find the equation of Nora's line for predicting the price of a used car given its age. Summarize the trend described by this equation.

d. Based on the line, for which car in the data set would the predicted value be farthest from the actual value? How can you tell?

e. What does the equation predict for the cost of a 10-year-old car? How close was the prediction using the line to the actual cost of the 10-year-old car in the data set? Given the context of the data set, do you think the difference between the predicted price and the actual price is large or small?

f. Is \$5,000 typical of the differences between predicted prices and actual prices for the cars in this data set? Justify your answer.

Lesson Summary

- A line can be used to represent the trend in a scatter plot.
- Evaluating the equation of the line for a value of the independent variable determines a value predicted by the line.
- A good line for prediction is one that goes through the middle of the points in a scatter plot and for which the points tend to fall close to the line.

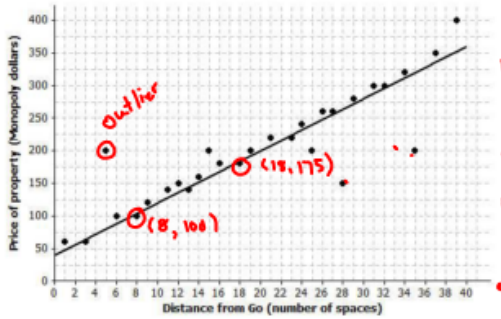
Problem Set

1. The Monopoly board game is popular in many countries. The scatter plot below shows the distance from "Go" to a property (in number of spaces moving from "Go" in a clockwise direction) and the price of the properties on the Monopoly board. The equation of the line is [redacted] where P represents the price (in Monopoly dollars) and x represents the distance (in number of spaces).

Distance from "Go" (number of spaces)	Price of Property (Monopoly dollars)
1	60
3	60
5	200
6	100
8	100
9	120
11	140
12	150
13	140
14	160
15	200
16	180
18	180
19	200

Distance from "Go" (number of spaces)	Price of Property (Monopoly dollars)
21	220
23	220
24	240
25	200
26	260
27	260
28	150
29	280
31	300
32	300
34	320
35	200
37	350
39	400

Price of Property Versus Distance from "Go" in Monopoly



$y = \text{rate} \times x + \text{extra}$

$y = \frac{75}{10}x + \text{extra}$

$y = 7.5x + 40$

rate means: (per) The price of property goes up 7.50 per space from Go.

extra means: When we are on Go, the price is $40.

a. Use the equation to find the difference (observed value−predicted value) for the most expensive property and for the property that is 35 spaces from "Go."

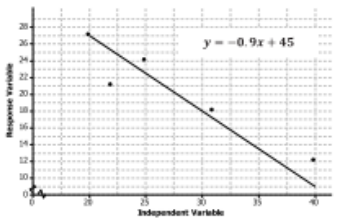
b. Five of the points seem to lie in a horizontal line. What do these points have in common? What is the equation of the line containing those five points?

c. Four of the five points described in part (b) are the railroads. If you were fitting a line to predict price with distance from "Go," would you use those four points? Why or why not?

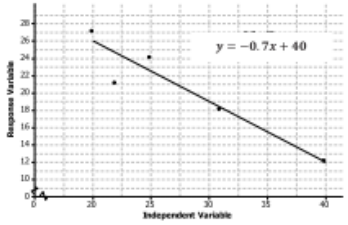
2. The table below gives the coordinates of the five points shown in the scatter plots that follow. The scatter plots show two different lines.

Data Point	Independent Variable	Response Variable
A	20	27
B	22	21
C	25	24
D	31	18
E	40	12

Line 1



Line 2



a. Find the predicted response values for each of the two lines.

Independent	Observed Response	Response Predicted by Line 1	Response Predicted by Line 2

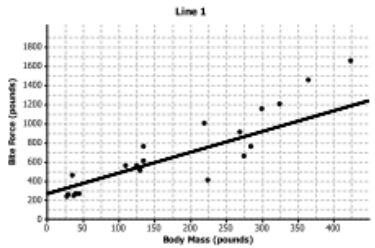
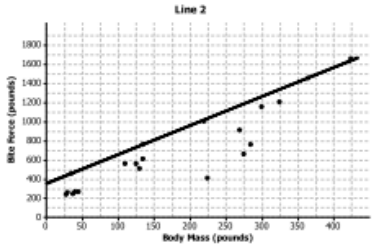
b. For which data points is the prediction based on Line 1 closer to the actual value than the prediction based on Line 2?

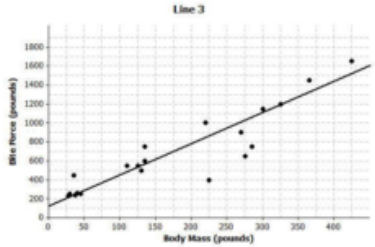
c. Which line (Line 1 or Line 2) would you select as a better fit? Explain.

3. The scatter plots below show different lines that students used to model the relationship between body mass (in pounds) and bite force (in pounds) for crocodilians.

   a. Match each graph to one of the equations below, and explain your reasoning. Let B represent bite force (in pounds) and W represent body mass (in pounds).

Equation 1	Equation 2	Equation 3
$B = 3.28W + 126$	$B = 3.04W + 351$	$B = 2.16W + 267$

Equation:	
Equation:	

Equation:	

b. Which of the lines would best fit the trend in the data? Explain your thinking.

4. Comment on the following statements:
   a. A line modeling a trend in a scatter plot always goes through the origin.
   b. If the response variable increases as the independent variable decreases, the slope of a line modeling the trend is negative.