Lesson 3: Existence and Uniqueness of Square Roots and Cube Roots

Classwork

Opening

The numbers in each column are related. Your goal is to determine how they are related, determine which numbers belong in the blank parts of the columns, and write an explanation for how you know the numbers belong there.

Find the Rule Part 1

1	1
2	4
3	9
9	81
11	121
15	225
7	49
10	100
12	144
13	169
m	m^2
$\sqrt{n}$	n

Find the Rule Part 2

1	1
2	8
3	27
5	125
6	216
11	1331
4	64
10	1,000
7	343
14	2,744
p	p^3
$\sqrt[3]{q}$	q

Exercises

Find the positive value of x that makes each equation true. Check your solution.

1. $x^2 = 169$

   a. Explain the first step in solving this equation.

   The first step is to square root both sides of the equation

   b. Solve the equation, and check your answer.

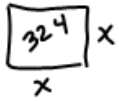
   $x^2 = 169$

   $\sqrt{x^2} = \sqrt{169}$

   $x = 13$

   $13^2 = 169$

2. A square-shaped park has an area of 324 yd^2. What are the dimensions of the park? Write and solve an equation.



   $x^2 = 324$

   $\sqrt{x^2} = \sqrt{324}$

   $x = 18$

   18 yd. × 18 yd

   $18^2 = 324$

3. $625 = x^2$

   $\sqrt{625} = \sqrt{x^2}$

   $25 = x$

   $25^2 = 625$

4. A cube has a volume of 27 in^3. What is the measure of one of its sides? Write and solve an equation.

   $x^3 = 27$

   $\sqrt[3]{x^3} = \sqrt[3]{27}$

   $x = 3$

   $3^3 = 27$

5. What positive value of x makes the following equation true: $x^2 = 64$? Explain.

$\sqrt{x} = \sqrt{64}$

$x = 8$ $\quad 8^2 = 64$

6. What positive value of x makes the following equation true: $x^3 = 64$? Explain.

$\sqrt[3]{x^3} = \sqrt[3]{64}$

$x = 4$ $\quad 4^3 = 64$

7. Find the positive value of x that makes the equation true: $x^2 = 256^{-1}$.

$x^2 = \frac{1}{256}$

$\sqrt{x^2} = \sqrt{\frac{1}{256}}$

$x = \frac{1}{16}$

8. Find the positive value of x that makes the equation true: $x^3 = 343^{-1}$.

$x^3 = \frac{1}{343}$

$\sqrt[3]{x^3} = \sqrt[3]{\frac{1}{343}}$

$x = \frac{1}{7}$

9. Is 6 a solution to the equation $x^2 - 4 = 5x$? Explain why or why not.

Lesson Summary

The symbol $\sqrt[n]{\ }$ is called a *radical*. An equation that contains that symbol is referred to as a *radical equation*. So far, we have only worked with square roots (i.e., $n = 2$). Technically, we would denote a positive square root as $\sqrt[2]{\ }$, but it is understood that the symbol $\sqrt{\ }$ alone represents a positive square root.

When $n = 3$, then the symbol $\sqrt[3]{\ }$ is used to denote the cube root of a number. Since $x^3 = x \cdot x \cdot x$, the cube root of x^3 is x (i.e., $\sqrt[3]{x^3} = x$).

The square or cube root of a positive number exists, and there can be only one positive square root or one cube root of the number.

Problem Set

Find the positive value of x that makes each equation true. Check your solution.

1. What positive value of x makes the following equation true: $x^2 = 289$? Explain.

$x =$

2. A square-shaped park has an area of $400\ \text{yd}^2$. What are the dimensions of the park? Write and solve an equation.

$x = 20$

3. A cube has a volume of $64\ \text{in}^3$. What is the measure of one of its sides? Write and solve an equation.

$x = 4$

4. What positive value of x makes the following equation true: $125 = x^3$? Explain.

$x = 5$

5. Find the positive value of x that makes the equation true: $x^2 = 441^{-1}$. $x = \frac{1}{21}$
   a. Explain the first step in solving this equation.
   b. Solve and check your solution.

6. Find the positive value of x that makes the equation true: $x^3 = 125^{-1}$. $x = \frac{1}{5}$

7. The area of a square is $196\ \text{in}^2$. What is the length of one side of the square? Write and solve an equation, and then check your solution. $x = 14$

8. The volume of a cube is $729\ \text{cm}^3$. What is the length of one side of the cube? Write and solve an equation, and then check your solution. $x = 9$

9. What positive value of x would make the following equation true: $19 + x^2 = 68$?

$x = 7$