Rational : Fraction

Divide : Top dog in the dog house

Lesson 8: The Long Division Algorithm

Terminating : stops
Bases 2, 5, 10

Repeating : repeats a pattern

Classwork

Example 1

Show that the decimal expansion of $\frac{26}{4}$ is 6.5.

$$\begin{array}{r} 6.5 \\ 4\,\overline{)\,26.0} \\ -24 \\ \hline 2\,0 \\ -2\,0 \\ \hline 0 \end{array}$$

Exploratory Challenge/Exercises 1–5

1.

a. Use long division to determine the decimal expansion of $\frac{142}{2}$.

$$\begin{array}{r} 71 \\ 2\,\overline{)\,142} \\ -14 \\ \hline 02 \\ -2 \end{array}$$

b. Fill in the blanks to show another way to determine the decimal expansion of $\frac{142}{2}$.

$142 = \underline{71} \times 2 + \underline{0}$

$\frac{142}{2} = \frac{\underline{71} \times 2 + \underline{0}}{2}$

$\frac{142}{2} = \frac{\underline{71} \times \cancel{2}}{\cancel{2}} + \frac{\underline{0}}{2}$

$\frac{142}{2} = \underline{71} + \frac{\underline{0}}{2}$

$\frac{142}{2} = \underline{71}$

c. Does the number $\frac{142}{2}$ have a finite or an infinite decimal expansion?

finite (terminating)

2.

a. Use long division to determine the decimal expansion of $\frac{142}{4}$.

Decimal adds a zero.

$$\begin{array}{r} 35.5 \\ 4\overline{)142.0} \\ -12 \\ \hline 22 \\ -20 \\ \hline 2\,0 \\ 2\,0 \\ \hline 0 \end{array}$$

b. Fill in the blanks to show another way to determine the decimal expansion of $\frac{142}{4}$.

$142 = \underline{35} \times 4 + \underline{2}$

$\frac{142}{4} = \frac{\underline{35} \times 4 + \underline{2}}{4}$

$\frac{142}{4} = \frac{\underline{35} \times 4}{4} + \frac{\underline{2}}{4}$

$\frac{142}{4} = \underline{35} + \frac{\underline{2}}{4}$

$\frac{142}{4} = \underline{35\frac{2}{4}}$

$35\frac{1}{2}$

Mixed Number uses remainder

c. Does the number $\frac{142}{4}$ have a finite or an infinite decimal expansion?

3.

a. Use long division to determine the decimal expansion of $\frac{142}{6}$.

$$\begin{array}{r} 23.\overline{6} \\ 6\,\overline{)\,142.0} \\ -12\downarrow \\ 22 \\ -18 \\ 40 \\ -36 \\ 4 \end{array}$$

b. Fill in the blanks to show another way to determine the decimal expansion of $\frac{142}{6}$.

$$142 = \underline{23} \times 6 + \underline{4}$$

$$\frac{142}{6} = \frac{\underline{23} \times 6 + \underline{4}}{6}$$

$$\frac{142}{6} = \frac{\underline{23} \times 6}{6} + \frac{\underline{4}}{6}$$

$$\frac{142}{6} = \underline{23} + \frac{\underline{4}\ \div 2}{6\ \div 2}$$

$$\frac{142}{6} = \underline{23\tfrac{2}{3}}$$

c. Does the number $\frac{142}{6}$ have a finite or an infinite decimal expansion?

$$23\tfrac{2}{3} = 23.\overline{6}$$

4.

a. Use long division to determine the decimal expansion of $\frac{142}{11}$. Repeat

11 cannot be made with a

. base 2, 5, or 10

b. Fill in the blanks to show another way to determine the decimal expansion of $\frac{142}{11}$.

$$142 = \_\_\_\_ \times 11 + \_\_\_\_$$

$$\frac{142}{11} = \frac{\_\_\_\_ \times 11 + \_\_\_\_}{11}$$

$$\frac{142}{11} = \frac{\_\_\_\_ \times 11}{11} + \frac{\_\_\_\_}{11}$$

$$\frac{142}{11} = \_\_\_\_ + \frac{\_\_\_\_}{11}$$

$$\frac{142}{11} = \_\_\_\_\_\_\_\_$$

c. Does the number $\frac{142}{11}$ have a finite or an infinite decimal expansion?

5. In general, which fractions produce infinite decimal expansions?

Exercises 6–10

repeat

6. Does the number $\frac{65}{13}$ have a finite or an infinite decimal expansion? Does its decimal expansion have a repeating pattern?

13 cannot be made with
a base 2, 5, 10.

7. Does the number $\frac{17}{11}$ have a finite or an infinite decimal expansion? Does its decimal expansion have a repeating pattern? Repeat

$\frac{1}{7} = 0.\overline{142857}$

$$\begin{array}{r} 0.142857 \\ 7\,\overline{)\,1.0000000} \\ -7 \\ \hline 30 \\ -28 \\ \hline 20 \\ -14 \\ \hline 60 \\ -56 \\ \hline 40 \\ -35 \\ \hline 50 \\ -49 \\ \hline 10 \end{array}$$

8. Is the number 0.212112111211112111112... rational? Explain. (Assume the pattern you see in the decimal expansion continues.)

No

There is no repeating block of numbers

9. Does the number $\frac{860}{999}$ have a finite or an infinite decimal expansion? Does its decimal expansion have a repeating pattern?

999 cannot be made with base 2, 5, or 10.

10. Is the number 0.12345678910111213141516171819202122 23... rational? Explain. (Assume the pattern you see in the decimal expansion continues.)

No

There is no repeating block of numbers.

Lesson Summary

A rational number is a number that can be written in the form $\frac{a}{b}$ for a pair of integers a and b with b not zero.

The long division algorithm shows that every rational number has a decimal expansion that falls into a repeating pattern. For example, the rational number 32 has a decimal expansion of $32.\overline{0}$, the rational number $\frac{1}{3}$ has a decimal expansion of $0.\overline{3}$, and the rational number $\frac{4}{11}$ has a decimal expansion of $0.\overline{45}$.

Problem Set

1. Write the decimal expansion of $\frac{7000}{9}$ as an infinitely long repeating decimal.

2. Write the decimal expansion of $\frac{6555555}{3}$ as an infinitely long repeating decimal.

3. Write the decimal expansion of $\frac{350000}{11}$ as an infinitely long repeating decimal.

4. Write the decimal expansion of $\frac{1200000}{37}$ as an infinitely long repeating decimal.

infinite

5. Someone notices that the long division of $2{,}222{,}222$ by 6 has a quotient of $370{,}370$ and a remainder of 2 and wonders why there is a repeating block of digits in the quotient, namely 370. Explain to the person why this happens.

6. Is the answer to the division problem number $10 \div 3.2$ a rational number? Explain.

stop

7. Is $\frac{3}{77}$ a rational number? Explain.

yes repeat

8. The decimal expansion of a real number x has every digit 0 except the first digit, the tenth digit, the hundredth digit, the thousandth digit, and so on, are each 1. Is x a rational number? Explain.