Lesson 2: Multiplication of Numbers in Exponential Form

Classwork

In general, if x is any number and m, n are positive integers, then

$$x^m \cdot x^n = x^{m+n}$$

because

$$x^m \times x^n = \underbrace{x \cdots x}_{m \text{ times}} \times \underbrace{x \cdots x}_{n \text{ times}} = \underbrace{x \cdots x}_{m+n \text{ times}} = x^{m+n}.$$

Exercise 1

$14^{23} \times 14^{8} =$

Exercise 2

$-72^{10} \times -72^{13} =$

Exercise 3

$5^{94} \times 5^{78} =$

Exercise 4

$-3^{9} \times -3^{5} =$

Exercise 5

Let a be a number.

$a^{23} \cdot a^{8} =$

Exercise 6

Let f be a number.

$f^{10} \cdot f^{13} =$

Exercise 7

Let b be a number.

$b^{94} \cdot b^{78} =$

Exercise 8

Let x be a positive integer. If $-3^{9} \times -3^{x} = -3^{14}$, what is x?

What would happen if there were more terms with the same base? Write an equivalent expression for each problem.

Exercise 9

$9^4 \times 9^6 \times 9^{13} =$

Exercise 10

$2^3 \times 2^5 \times 2^7 \times 2^9 =$

Can the following expressions be simplified? If so, write an equivalent expression. If not, explain why not.

Exercise 11

$6^5 \times 4^9 \times 4^3 \times 6^{14} =$

Exercise 12

$-4^{\ 2} \cdot 17^5 \cdot -4^{\ 3} \cdot 17^7 =$

Exercise 13

$15^2 \cdot 7^2 \cdot 15 \cdot 7^4 =$

Exercise 14

$2^4 \times 8^2 = 2^4 \times 2^6 =$

Exercise 15

$3^7 \times 9 = 3^7 \times 3^2 =$

Exercise 16

$5^4 \times 2^{11} =$

Exercise 17

Let x be a number. Simplify the expression of the following number:

$2x^3 \quad 17x^7 \quad =$

Exercise 18

Let a and b be numbers. Use the distributive law to simplify the expression of the following number:

$a \ a + b \ =$

Exercise 19

Let a and b be numbers. Use the distributive law to simplify the expression of the following number:

$b\ a+b\ =$

Exercise 20

Let a and b be numbers. Use the distributive law to simplify the expression of the following number:

$a+b\ \ a+b\ =$

In general, if x is nonzero and m, n are positive integers, then

when dividing exponents with the same base subtract the powers

$$\frac{x^m}{x^n} = x^{m-n}, \text{ if } m > n.$$

Exercise 21

$\frac{7^9}{7^6} = \frac{7\cdot7\cdot7\cdot7\cdot7\cdot7\cdot7\cdot7\cdot7}{7\cdot7\cdot7\cdot7\cdot7\cdot7} = 7^3$

Exercise 23

$\frac{\left(\frac{8}{5}\right)^9}{\left(\frac{8}{5}\right)^2} = \left(\frac{8}{5}\right)^{9-2} = \left(\frac{8}{5}\right)^7$

Exercise 22

$\frac{(-5)^{16}}{(-5)^7} = (-5)^{16-7} = (-5)^9$

Exercise 24

$\frac{13^5}{13^4} = 13^{5-4} = 13$

Exercise 25

Let a, b be nonzero numbers. What is the following number?

$\frac{\frac{a}{b}^{9}}{\frac{a}{b}^{2}} = \left(\frac{a}{b}\right)^{7}$

Exercise 26

Let x be a nonzero number. What is the following number?

$\frac{x^5}{x^4} =$ X

Can the following expressions be simplified? If yes, write an equivalent expression for each problem. If not, explain why not. ✱ Collect the matching bases

Exercise 27

$\frac{2^7}{2^4} = \frac{2^7}{2^4} = 2^3$

$(2^3)^2$

Exercise 29

$\frac{3^5 \cdot 2^8}{3^2 \cdot 2^3} = 3^3 \cdot 2^5$

Exercise 28

$\frac{3^{23}}{27} = \frac{3^{23}}{3^3} = 3^{20}$

Exercise 30

$\frac{(-2)^7 \cdot 95^5}{(-2)^5 \cdot 95^4} = (-2)^2 \cdot 95$

$= 4 \cdot 95 = 380$

when dividing large expressions, look for the matching bases and keep the base where the bigger power is. * Simplify coefficients

Exercise 31

Let x be a number. Simplify the expression of each of the following numbers:

a. $\frac{5}{x^3}(3x^8) = \frac{15x^8}{x^3} = 15x^5$

b. $\frac{5}{x^3}(-4x^6) = \frac{-20x^6}{x^3} = -20x^3$

c. $\frac{5}{x^3}(11x^4) = \frac{55x^4}{x^3} = 55x$

a.) $\frac{1\ 20\ a^5\ b^2\ c^7}{2\ 40\ a^2\ b^5\ c^3} = \frac{1\ a^3\ c^4}{2\ b^3}$

$= \boxed{\frac{a^3 c^4}{2b^3}}$

Exercise 32

Anne used an online calculator to multiply $2{,}000{,}000{,}000 \times 2{,}000{,}000{,}000{,}000$. The answer showed up on the calculator as $4e + 21$, as shown below. Is the answer on the calculator correct? How do you know?

$2 \times 2 = 4$

"4 with 21 zeros"

4×10^{21}

$$\frac{x^2 y}{x y^3} = \frac{x}{y^2}$$

$$\frac{x^{20}}{x^{7}} = x^{13}$$

$$\frac{x^2 \cdot x^3}{x^4} = \frac{x^5}{x^4} = x$$

$$\frac{m^{10}}{m^2 \cdot m^6} = \frac{m^{10}}{m^8} = m^2$$

$$\frac{12x^{3-1}y^{2}}{24x^{4-2}y} = \frac{1x^{2}}{2y^{2}}$$

$$= \frac{x^{2}}{2y^{2}}$$

$$\frac{12\,m^{7}\,n^{5}}{m^{2}} = 12m^{5}n^{5}$$

$$\frac{14x^{15}y^{-1}}{3\cancel{12}\,\cancel{y}} = \frac{x^{14}}{3y}$$

Problem Set

1. A certain ball is dropped from a height of x feet. It always bounces up to $\frac{2}{3}x$ feet. Suppose the ball is dropped from 10 feet and is caught exactly when it touches the ground after the 30^{th} bounce. What is the total distance traveled by the ball? Express your answer in exponential notation.

Bounce	Computation of Distance Traveled in Previous Bounce	Total Distance Traveled (in feet)
1		
2		
3		
4		
30		
n		

2. If the same ball is dropped from 10 feet and is caught exactly at the highest point after the 25^{th} bounce, what is the total distance traveled by the ball? Use what you learned from the last problem.

3. Let a and b be numbers and $b \neq 0$, and let m and n be positive integers. Simplify each of the following expressions as much as possible:

$-19^{5} \cdot -19^{11} =$	$2.7^{5} \times 2.7^{3} =$
$\frac{7^{10}}{7^{3}} =$	$\left(\frac{1}{5}\right)^{2} \cdot \left(\frac{1}{5}\right)^{15} =$
$\left(-\frac{9}{7}\right)^{m} \cdot \left(-\frac{9}{7}\right)^{n} =$	$\frac{ab^{3}}{b^{2}} =$

4. Let the dimensions of a rectangle be $(4 \times 871209^{5} + 3 \times 49762105)$ ft. by $7 \times 871209^{3} - 49762105^{4}$ ft. Determine the area of the rectangle. No need to expand all the powers.

5. A rectangular area of land is being sold off in smaller pieces. The total area of the land is 2^{15} square miles. The pieces being sold are 8^{3} square miles in size. How many smaller pieces of land can be sold at the stated size? Compute the actual number of pieces.