

Zero Exponent

**Anything to the zero power equals 1**

$$a.) \frac{2}{2^3} = 2^{3-3} = 2^0 = 1$$

$$= \frac{8}{8}$$

$$b.) (x^2 y^5 z)^0 = 1$$

$$c.) 2x^2 y^0 = 2x^2 \cdot 1 = 2x^2$$



$2^3 = 8$	$\div 2$	
$2^2 = 4$	$\div 2$	
$2^1 = 2$	$\div 2$	
$2^0 = 1$	$\div 2$	
$2^{-1} = \frac{1}{2}$	$\div 2 = \frac{1}{2^1}$	
$2^{-2} = \frac{1}{4}$	$\div 2 = \frac{1}{2^2}$	Simplified
$2^{-3} = \frac{1}{8}$	$\div 2 = \frac{1}{2^3}$	

Not Simplified

## Lesson 5: Negative Exponents and the Laws of Exponents

## Classwork

**Definition:** For any positive number  $x$  and for any positive integer  $n$ , we define  $x^{-n} = \frac{1}{x^n}$ .

Note that this definition of negative exponents says  $x^{-1}$  is just the reciprocal  $\frac{1}{x}$  of  $x$ .

As a consequence of the definition, for a positive  $x$  and all integers  $b$ , we get

$$x^{-b} = \frac{1}{x^b}$$

**Exercise 1**

Verify the general statement  $x^{-b} = \frac{1}{x^b}$  for  $x = 3$  and  $b = -5$ .

**Exercise 2**

What is the value of  $(3 \times 10^{-2})^2$ ?

$$= (3 \times \frac{1}{10^2})^2$$

$$= \frac{3}{100} = 0.03$$

**Fully Simplified means**

one of each letter no zero e

To make exponents positive, move them to the denominator (either up or down)

**Exercise 3**

What is the value of  $(3 \times 10^{-5})$ ?

**Exercise 4**

Write the complete expanded form of the decimal 4.728 in exponential notation.

For Exercises 5–10, write an equivalent expression, in exponential notation, to the one given and simplify as much as possible.

**Exercise 5**

$$5^{-3} = \frac{1}{5^3} = \frac{1}{125}$$

**Exercise 8**

Let  $x$  be a nonzero number.

$$x^{-3} = \frac{1}{x^3}$$

**Exercise 6**

$$\frac{1}{8^9} = 8^{-9}$$

**Exercise 9**

Let  $x$  be a nonzero number.

$$\frac{1}{x^9} =$$

**Exercise 7**

$$3 \cdot 2^{-4} = 3 \cdot \frac{1}{2^4} = \frac{3}{16}$$

**Exercise 10**

Let  $x, y$  be two nonzero numbers.

$$xy^{-4} = x \cdot \frac{1}{y^4} = \frac{x}{y^4}$$

We accept that for positive numbers  $x$ ,  $y$  and all integers  $a$  and  $b$ ,

$$x^a \cdot x^b = x^{a+b}$$

$$(x^b)^a = x^{ab}$$

$$(xy)^a = x^a y^a$$

We claim:

$$\frac{x^a}{x^b} = x^{a-b} \quad \text{for all integers } a, b$$

$$\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a} \quad \text{for any integer } a$$

#### Exercise 11

$$\frac{19^2}{19^5} =$$

#### Exercise 12

$$\frac{17^{16}}{17^{-3}} =$$

#### Exercise 13

If we let  $b = -1$  in (11),  $a$  be any integer, and  $y$  be any positive number, what do we get?

$$\frac{2x^{-2}y^3}{4z^{-4}} = \frac{1y^3z^4}{2x^2}$$

$$\frac{5a^2b^0c^{-3}}{25d^7e^4} = \frac{a^2e^4}{5c^3d^5}$$

## Exercise 14

Show directly that  $\left(\frac{7}{5}\right)^{-4} = \frac{7^{-4}}{5^{-4}} = \frac{5^4}{7^4}$

$$\begin{aligned} a) \quad & \left(\frac{xy^{-2}}{ab^3}\right)^{-3} \\ & = \left(\frac{ab^3}{xy^{-2}}\right)^3 \\ & = \frac{a^3b^9}{x^3y^{-6}} \\ & = \frac{a^3b^9y^6}{x^3} \end{aligned}$$

$$\begin{aligned} & = \frac{x^{-3}y^6}{a^{-3}b^{-9}} \\ & = \frac{a^3b^9y^6}{x^3} \end{aligned}$$

When a large quotient is brought to a negative power, you can flip the entire expression to make the exponent positive.

## Problem Set

1. Compute:  $3^3 \times 3^2 \times 3^1 \times 3^0 \times 3^{-1} \times 3^{-2} =$   
 Compute:  $5^2 \times 5^{10} \times 5^8 \times 5^0 \times 5^{-10} \times 5^{-8} =$   
 Compute. For a nonzero number,  $a$ :  $a^m \times a^n \times a^1 \times a^{-n} \times a^{-m} \times a^{-1} \times a^0 =$
2. Without using (10), show directly that  $(17.6^{-1})^8 = 17.6^{-8}$ .
3. Without using (10), show (prove) that for any whole number  $n$  and any positive number  $y$ ,  $(y^{-1})^n = y^{-n}$ .
4. Show directly without using (13) that  $\frac{2.8^{-5}}{2.8^7} = 2.8^{-12}$ .