CONCEPT DEVELOPMENT

Mathematics Assessment Project

CLASSROOM CHALLENGES

A Formative Assessment Lesson

Estimating Length Using Scientific Notation

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley

Beta Version

Estimating Length Using Scientific Notation

MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students are able to:

- Estimate lengths of everyday objects.
- Convert between decimal and scientific notation.
- Make comparisons of the size of numbers expressed in both decimal and scientific notation.

STANDARDS ADDRESSED

This lesson relates to the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics*:

8-EE: Work with radicals and integer exponents.

This lesson also relates to the following *Standards for Mathematical Practice* in the *Common Core State Standards for Mathematics*:

2 Reason abstractly and quantitatively.

7. Look for and make use of structure.

INTRODUCTION

The lesson unit is structured in the following way:

- Before the lesson, students work individually on an assessment task that is designed to reveal their current understanding and difficulties. You then review their responses and create questions for students to consider in order to improve their work.
- After a whole-class introduction, students work in small groups on a collaborative task matching measurements expressed in decimal and scientific notation. They then match these measurements to everyday objects.
- Students then make comparisons between the objects by investigating the size of one object in relation to another. To end the lesson there is a whole-class discussion.
- In a follow-up lesson, students work alone again on a similar task to the introductory task.

MATERIALS REQUIRED

- Each student will need a copy of the assessment tasks *Size It Up* and *Resize It Up!,* a mini-whiteboard, a pen, and an eraser.
- Each small group of students will need *Card Set A: Measurements*, *Card Set B: Objects, Card Set C: Comparisons (all cards cut out before the lesson),* a large sheet of paper, and a glue stick.
- There is a projector resource to support whole-class discussions.

TIME NEEDED

15 minutes before the lesson for the assessment task, a 75-minute lesson, and 15 minutes in a follow-up lesson (or for homework). Timings given are only approximate. Exact timings will depend on the needs of the class.

BEFORE THE LESSON

Assessment task: *Size It Up* (15 minutes)

Ask the students to complete this task, in class or for homework, a day or more before the formative assessment lesson. This will give you an opportunity to assess the work and to find out the kinds of difficulties students have with it. You will then be able to target your help more effectively in the follow-up lesson.

Give each student a copy of the task *Size It Up*.

> *Read through the questions and try to answer them as carefully as you can.*

It is important that, as far as possible, students are allowed to answer the questions without your assistance.

Students should not worry if they cannot understand or do everything because in the next lesson they will work on a similar task, which should help them. Explain that by the end of the next lesson, students should be able to answer questions such as these confidently. This is their goal.

Size It Up

1. The table contains seven measurements written in decimal and scientific notation.
   (a) Complete the table so that each measurement is written in *both* decimal *and* scientific notation.
   (b) In the last column, rank the measurements in order of size.
   (1 = smallest, 2 = next smallest, and so on up to 7 = largest)

Decimal Notation		Scientific Notation	Rank 1 = smallest 7 = largest
	=	1×10^{-2} m	
0.004 m	=		
200 m	=		
	=	8×10^{5} m	
40,000,000 m	=		
40 m	=		
	=	8×10^{-4} m	

2 (a) Complete the following statement using two numbers in *decimal notation* from the table.

.... × 4000 = ...

2 (b) Complete the following statement using two numbers in *scientific notation* from the table.

.... × 50,000 = ...

Assessing students' responses

Collect students' responses to the task, and note down what their work reveals about their current levels of understanding and their different approaches.

We suggest that you do not score students' work. The research shows that this will be counterproductive, as it will encourage students to compare their scores and will distract their attention from what they can do to improve their mathematics.

Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some suggestions for these are given on the next page. These have been drawn from common difficulties observed in trials of this unit.

We suggest that you write a list of your own questions, based on your students' work, using the ideas that follow. We recommend that you write questions on each individual student's work. If you do not have time to do this, select a few questions that will be of help to the majority of students. These can be written on the board at the start of the follow-up lesson.

Common issues:	**Suggested questions and prompts:**
Converts measurements from scientific to decimal notation incorrectly For example: Writes 1×10^{-2} m as 0.001 m.	• What does $\times 10^{-1}$ mean? What does $\times 10^{-2}$ mean? • How can you check your answer is correct?
Does not remember the requirements for scientific notation For example: Writes 40,000,000 m as 40×10^{6} m.	• Have any of these numbers been written in scientific notation? 20×10^{-2}, 0.6×10^{4}, 5×10^{3} How do you know? •
Does not fully understand the effects of negative exponents For example: The student assumes that 8×10^{-4} is bigger than 1×10^{-2}.	• Is 10^{-4} bigger than 10^{-2}? How do you know? • In scientific notation, which has a bigger effect on the size of a number, the coefficient or the exponent? Explain. • Is 9×10^{1} bigger than 1×10^{2}? What is the difference between the two numbers? • Is 9×10^{-2} bigger than 1×10^{-1}? What is the difference between the two numbers?
Fails to complete the size rankings for the measurements	• Which measurement is the smallest? Which measurements is the biggest? How do you know?
Makes incorrect assumptions when comparing measurements For example: The student assumes that question 2 cannot be completed as the coefficients in the scientific notation are limited to 1, 2, 4 and 8 and so you can't multiply any of these by either 50 or 4000 to get another listed coefficient.	• Can you use the decimal notation to help you to complete the second question? • Numbers in scientific notation include an integer power of ten. What effect does this have?
Completes the task The student needs an extension task.	• Choose two measurements that you have not yet used in Q2. Can you work out how much bigger one measurement is than the other? • How much bigger is the measurement you have ranked as 2 than rank 1? Can you compare your other rankings in this way?

SUGGESTED LESSON OUTLINE

Whole-class introduction (20 minutes)

Give each student a mini-whiteboard, pen and eraser.

Display slide P-1 of the projector resource:

What's the Number?
$30{,}000 \times 10^{-1}$
3×10^{3}
0.3×10^{4}

These three expressions all represent the same number.
Write the number on your whiteboard [3000.]

Which of the expressions did you use to work out the number?
Why did you choose this expression?

Did you check that the other expressions gave the same number?

It is likely that students will use the second representation to figure out the number. Ask students to justify why all three expressions are equal.

Display slide P-2 of the projector resource, which contains the same numbers as slide 1.

Which of these three expressions is written in scientific notation?

Some students may assume that all three expressions are in scientific notation. Spend some time discussing student responses and address any misconceptions that arise. Students should have a clear understanding that 3×10^3 is the only expression written using scientific notation and be able to justify why the other two representations are not scientific notation.

A number is written in scientific notation when it is of the form $a \times 10^n$, where $1 \le a < 10$ and n is an integer.

(Note that, when the decimal number is negative, $-10 < a \le -1$, but in this lesson we focus on physical lengths, which are greater than zero.)

Display slide P-3 of the projector resource:

Which is Greater?
8×10^{-3}
4×10^{-1}

Write on your whiteboard the number you think is the greater of the two.

If students' opinions are divided, ask students to pair up with another student with the same choice.

In your pair, work out how much greater your chosen number is than the other number.

Once students have had a chance to complete this calculation, choose a few pairs of students to feedback their choice and any calculations they have carried out. A common misconception is to ignore the exponent and assume that 8×10^{-3} is twice as big as 4×10^{-1} due to the fact that $4 \times 2 = 8$. If all students have correctly identified the greater of the two numbers, it may be appropriate to explore the different methods used to compare the two numbers and address any differences in the comparison of sizes.

Collaborative activity: *Card Set A: Measurements* (10 minutes)

Organize the class into groups of two or three students.

Give each group *Card Set A: Measurements*.

Explain how students are to work collaboratively. Slide P-4 of the projector resource summarizes these instructions.

Take turns to match a card in decimal notation with a card in scientific notation.

Each time you match a pair of cards, explain your thinking clearly and carefully. Place your cards side by side on your desk, not on top of one another, so that everyone can see them.

Partners should either agree with the explanation, or challenge it if it is not clear, or not complete.

It is important that everyone in the group understands the matching of each card.

You should find that two cards do not have a match. Write the alternative notation for these measurements on the blank cards to produce a pair.

If students are struggling to get started, it may be appropriate to demonstrate a correct match with the whole class. It may also be helpful for students to divide the cards into separate piles dependent on notation, before looking for matches.

As students work, you have two tasks: to note student approaches to the task and to support student reasoning.

Note student approaches to the task

Listen and watch students carefully. In particular, notice how students make a start on the task, where they get stuck, and how they overcome any difficulties. Do students prefer to convert from scientific to decimal notation or the other way around? How are they working out the value of the exponent? You can use this information to focus a whole-class discussion towards the end of the lesson.

Support student reasoning

Try not to make suggestions that move students towards a particular matching. Instead, ask questions to help students to reason together. You may want to use some of the questions in the *Common issues* table. If you find one student within the group has matched a particular pair of cards, challenge another student in the group to explain the match.

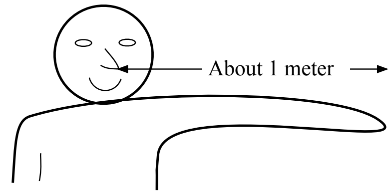
Chen matched these two cards. Christie, why does Chen think these two cards match?

When completing the blank measurement cards, some students may assume that for negative exponents, the magnitude of the exponent corresponds to the number of zeros after the decimal point (for example, assuming that $1.2 \times 10^{-2} = 0.0012$ rather than 0.012.) Encourage students to explain their reasoning and use this process as a method for checking their work.

Collaborative activity: *Card Set B: Objects* (15 minutes)

When students have finished matching the measurement cards, give each group *Card Set B: Objects*.

Students may already have noticed that each measurement card has a 'm' at the end. Ask students to suggest what this represents and clarify that all of the measurements are in meters.



For the next part of the lesson students will be matching up an everyday object from *Card Set B* with the appropriate measurement cards from *Card Set A*. It might be helpful to illustrate the rough size of a meter. Explain to students that the distance from your nose to a fingertip is about one meter.

Ask students to match the objects in *Card Set B* with the corresponding paired measurements from *Card Set A*.

If students are struggling, suggest putting the object cards in order of size first and then matching them up to the measurements cards based on order of size.

As students complete all matches, ask them to check their work against that of a neighboring group.

If there are differences between your matching, ask for an explanation. If you still don't agree, explain your own thinking.

You must not move the cards for another group. Instead, explain why you disagree with a particular match and explain which cards you think need to be changed.

Slide P-5 of the projector resource summarizes these instructions.

Collaborative activity: *Card Set C: Comparisons* (20 minutes)

When students are satisfied with their matches, give each group *Card Set C: Comparisons*, a glue stick, and a large sheet of paper for making a poster:

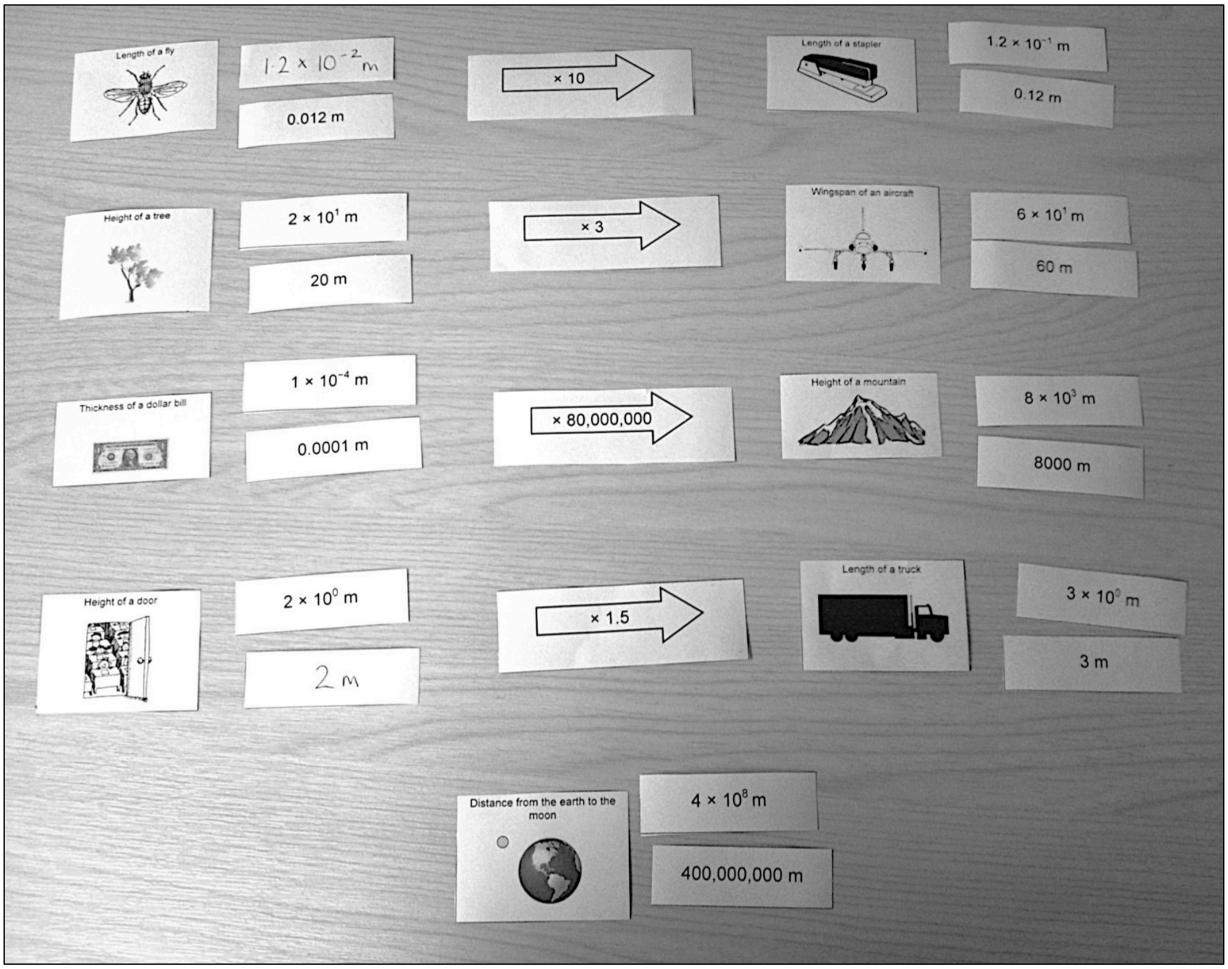
You are now to place an arrow card between two sets of measurements/objects to show how much bigger one object is than another.

For example, you may choose to start with the arrow that has "×3" on it. Are there two objects that have measurements such that the measurement for one of the objects multiplied by 3 gives the measurement for the other object?

Alternatively, you may want to start with two measurements/objects and see if there is an arrow that describes how much bigger one object is compared to the other. There are some blank arrow cards for you to use if needed.

Once you have linked a pair of measurements/objects, stick the cards down on your poster and see if you can compare another pair of objects. Do this for as many pairs as possible.

Their finished poster may look something like this:



Students will be able to make a maximum of four paired comparisons with one set of object/measurement cards left over. These can be used in the extension activity if time allows. They will not need all of the arrow cards.

As students work on the task, encourage them to check their work. They may find it helpful to think about the matched objects to check whether or not their comparison is realistic, before carrying out any calculations:

Roughly how many times bigger is the height of a tree than the height of a door? Is this a realistic comparison? Can you now check it mathematically?

Students should be encouraged to justify the comparisons made and describe their methods:

Can you explain why you placed this arrow here?

In this task, do you find it easier to use the decimal notation card or the scientific notation card? Why?

When students have completed as many comparisons as possible and stuck them down, ask them to stick down any remaining matched up object/measurement cards on their poster as well.

Extension activity

If a group successfully completes four comparisons, ask them to compare their remaining object with an object of their own choice. They need to decide on an object and its measurement and state how many times bigger it is. They can either select one of the unused arrow cards or write their own.

Whole-class discussion (10 minutes)

Organize a discussion about what has been learned and encourage students to explore the different methods used. It is likely that each poster will be slightly different, depending on the comparisons made.

Michael, explain how you matched one arrow card with two measurement/object cards.

Did anyone else complete the same comparison?

In addition to asking for a variety of methods, pursue the theme of listening and comprehending others' methods by asking students to rephrase others' reasoning.

Can someone else put that into your own words?

Does anyone disagree with this comparison?

There will not be time to discuss all the comparisons, but a good coverage of measurements should be encouraged.

Follow-up lesson: Reviewing the assessment task (15 minutes)

Give each student a copy of the review task, *Resize It Up!*, and their original scripts from the assessment task, *Size It Up*. If you have not added questions to individual pieces of work, then write your list of questions on the board. Students should select from this list only those questions they think are appropriate to their own work.

Read through your scripts from Size It Up, and the questions [on the board/written on your script.] Think about what you have learned.

Now look at the new task sheet, Resize It Up! Can you use what you have learned to answer these questions?

SOLUTIONS

Assessment Task: *Size It Up*

1.

Decimal Notation	Scientific Notation	Rank order
0.01 m	1×10^{-2} m	3
0.004 m	4×10^{-3} m	2
200 m	2×10^{2} m	5
800,000 m	8×10^{5} m	6
40,000,000 m	4×10^{7} m	7
40 m	4×10^{1} m	4
0.0008 m	8×10^{-4} m	1

2 (a) There are two alternatives:

0.01 m	$\times$ 4000	=	40 m
200 m	$\times$ 4000	=	800,000 m

2 (b) There are two alternatives:

8×10^{-4} m	$\times$ 50,000	=	4×10^{1} m
4×10^{-3} m	$\times$ 50,000	=	2×10^{2} m

Assessment Task: *Resize It Up*!

1.

Decimal Notation	Scientific Notation	Rank order
6 m	6×10^{0} m	4
120,000 m	1.2×10^{5} m	7
0.0012 m	1.2×10^{-3} m	2
300 m	3×10^{2} m	5
0.6 m	6×10^{-1} m	3
0.0009 m	9×10^{-4} m	1
6,000 m	6×10^{3} m	6

2 (a) There are two alternatives:

300 m	$\times 20$	=	6,000 m
6,000 m	$\times 20$	=	120,000 m

2 (b) There are two alternatives:

6×10^{-1} m	$\times 500$	=	3×10^{2} m
1.2×10^{-3} m	$\times 500$	=	6×10^{-1} m

Collaborative activity

The measurements in **bold** correspond to blank cards.

Scientific Notation	Decimal Notation	Object
1×10^{-4} m	0.0001 m	Thickness of a dollar bill
1.2×10^{-2} m	0.012 m	Length of a fly
1.2×10^{-1} m	0.12 m	Length of a stapler
2×10^{0} m	**2 m**	Height of a door
3×10^{0} m	3 m	Length of a truck
2×10^{1} m	20 m	Height of a tree
6×10^{1} m	60 m	Wingspan of an aircraft
8×10^{3} m	8000 m	Height of a mountain
4×10^{8} m	400,000,000 m	Distance from the earth to the moon

There are numerous comparisons that can be made. For example:

Length of a truck 3×10^{0} m $= 3$ m → $\times 20$ → Wingspan of an aircraft 6×10^{1} m $=$ 60 m

Thickness of a dollar bill 1×10^{-4} m $=$ 0.0001 m → $\times$ 80,000,000 → Height of a mountain 8×10^{3} m $=$ 8000 m

Some of the arrows will be redundant depending on the comparisons made.

Size It Up

1. The table contains seven measurements written in decimal and scientific notation.
   (a) Complete the table so that each measurement is written in *both* decimal *and* scientific notation.
   (b) In the last column, rank the measurements in order of size.
   (1 = smallest, 2 = next smallest, and so on up to 7 = largest)

Decimal Notation		Scientific Notation	Rank 1 = smallest 7 = largest
	=	1×10^{-2} m	
0.004 m	=		
200 m	=		
	=	8×10^{5} m	
40,000,000 m	=		
40 m	=		
	=	8×10^{-4} m	

2 (a) Complete the following statement using two numbers in *decimal notation* from the table.

------------------------ × 4000 = ----------------------------

2 (b) Complete the following statement using two numbers in *scientific notation* from the table.

---------------------- × 50,000 = ------------------------------

Card Set A: Measurements

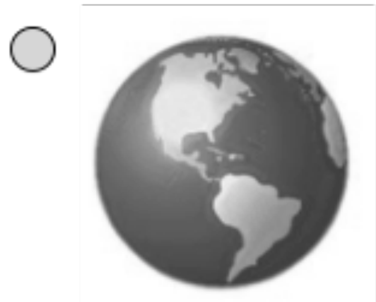
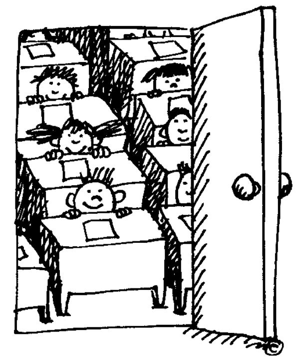
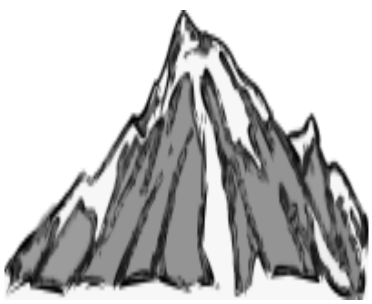
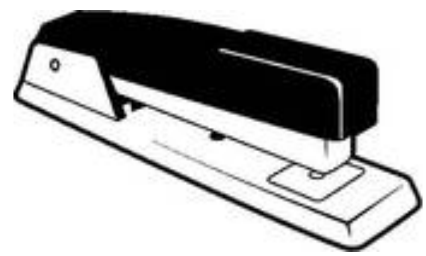
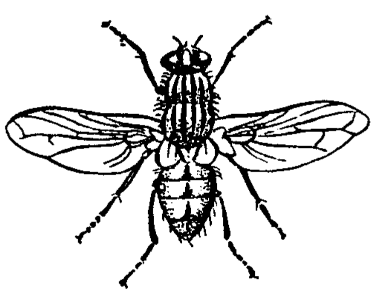
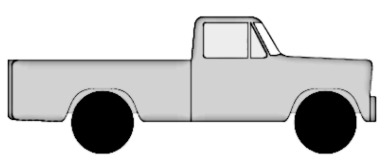
Decimal notation

20 m	**60 m**	**0.012 m**
0.12 m	**3 m**	**8000 m**
400,000,000 m	**0.0001 m**	

Scientific notation

2×10^{0} m	1×10^{-4} m	8×10^{3} m
3×10^{0} m	2×10^{1} m	4×10^{8} m
6×10^{1} m	1.2×10^{-1} m	

Card Set B: Objects

Height of a tree 	Distance from the earth to the moon 	Height of a door 
Height of a mountain 	Thickness of a dollar bill 	Length of a stapler 
Length of a fly 	Length of a truck 	Wingspan of an aircraft 

Card Set C: Comparisons

× 1200	× 1.5
× 20,000	× 10
× 20	× 400
× 3	× 80,000,000
× 4000	× 120
× 30	× 30,000

Resize It Up!

1. The table contains seven measurements written in decimal and scientific notation.
   (a) Complete the table so that each measurement is written in *both* decimal *and* scientific notation.
   (b) In the last column, rank the measurements in order of size.
   (1 = smallest, 2 = next smallest, and so on up to 7 = largest)

Decimal Notation		Scientific Notation	Rank 1 = smallest 7 = largest
	=	6×10^0 m	
120,000 m	=		
0.0012 m	=		
	=	3×10^2 m	
0.6 m	=		
	=	9×10^{-4} m	
6000 m	=		

2 (a) Complete the following statement using two numbers in *decimal notation* from the table.

------------------------ × 20 = ----------------------------

2 (b) Complete the following statement using two numbers in *scientific notation* from the table.

------------------------ × 500 = ----------------------------

What's the Number?

$$30{,}000 \times 10^{-1}$$

$$3 \times 10^{3}$$

$$0.3 \times 10^{4}$$

Scientific Notation or Not?

$$30{,}000 \times 10^{-1}$$

$$3 \times 10^{3}$$

$$0.3 \times 10^{4}$$

Which is Greater?

8×10^{-3}

4×10^{-1}

Matching *Card Set A*

1. Take turns to match a card in scientific notation with a card in decimal notation.

2. Each time you match a pair of cards explain your thinking clearly and carefully. Place your cards side by side on your desk, not on top of one another, so that everyone can see them.

3. Partners should either agree with the explanation, or challenge it if it is not clear or not complete.

4. It is important that everyone in the group understands the matching of each card.

5. You should find that two cards do not have a match. Write the alternative notation for these measurements on the blank cards to produce a pair.

Matching *Card Set B*

1. Match the objects in *Card Set B* with the corresponding paired measurements from *Card Set A*. You may want to put the objects in size order first to help you.

2. It is important that everyone in the group agrees on each match.

3. When you have finished matching the cards, check your work against that of a neighboring group.

4. If there are differences between your matching, ask for an explanation. If you still don't agree, explain your own thinking.

5. You must not move the cards for another group. Instead, explain why you disagree with a particular match and explain which cards you think need to be changed.

Mathematics Assessment Project

CLASSROOM CHALLENGES

This lesson was designed and developed by the
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It was refined on the basis of reports from teams of observers led by
David Foster, Mary Bouck, and Diane Schaefer
based on their observation of trials in US classrooms
along with comments from teachers and other users.

This project was conceived and directed for
MARS: Mathematics Assessment Resource Service
by
Alan Schoenfeld, Hugh Burkhardt, Daniel Pead, and Malcolm Swan
and based at the University of California, Berkeley

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